Simple Harmonic Oscillator

Equipment:

- No special safety equipment is required for this lab.
- Computer with modern HTML5 web browser.
- https://phet.colorado.edu/en/simulation/masses-and-springs

Introduction

Imagine a spring that is suspended from a support. When no mass is attached at the end of the spring, it has a length L (called its rest length). If a mass is added to the spring, its length increases by ΔL. The equilibrium position of the mass is now a distance $L + \Delta L$ from the spring's support

What happens if the mass is pulled down a small distance ***A*** from the equilibrium position? The spring exerts a restoring force, $F = -kx$, where ***x*** is the distance the spring is pulled down and ***k*** is the force constant of the spring. The negative sign indicates that the force points opposite of the direction of the displacement of the mass. The restoring force causes the mass to oscillate or move up and down within a range of ***A*** from the equilibrium. The distance A or maximum displacement from the equilibrium is called ***an amplitude*** of oscillation. The period of oscillation for simple harmonic motion depends on the mass and the force constant of the spring.

We expect that the frequency of the oscillations will be found from:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, f is the frequency in hertz (Hz), k is the spring constant in N/m, and m is the mass attached to the spring, in kg. Sometimes the angular frequency is used. It is $\omega = 2\pi f$.

Another measure of the oscillations is the period. This is the time for one oscillation. It is:

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

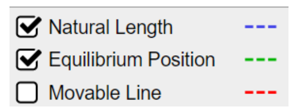
Note that the amplitude of oscillation is not present in the formulas above and, therefore, it has no effect on the period and frequency of oscillation.

Objective

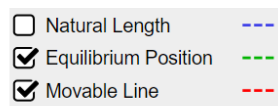
- *To verify the dependence of a period of a spring-mass system acting as a simple harmonic* *oscillator on mass, spring constant, and amplitude.*

Part #1. Simulation of a Spring-Mass System

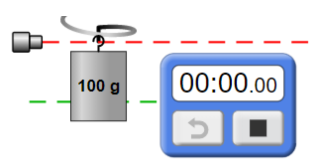
1. Open the PhET Masses and Springs simulation: https://phet.colorado.edu/en/simulation/masses-and-springs. Use the first version of the simulation, called Intro.
2. Hang a 100 g mass from either spring. When you release it, it will probably oscillate. Press the stop sign near the top of the spring to stop it at the equilibrium position.
3. Turn on the Natural Length and the Equilibrium Position lines. Notice that the mass has pulled the spring downward from its natural length. Also note that the green Equilibrium Position line goes through the center of the hook. This is the reference point on the object.



4. Measure the equilibrium stretch of the spring using the simulated ruler tool (units in cm). Use equilibrium stretch to calculate the spring constant. (Hint: $k\ y = m\ g$ and don't forget to convert units). Use the spring constant to calculate the "predicted frequency" with the equation in the Introduction.
5. Turn off the blue Natural Length line, and turn on the red Movable Line.



6. **<u>Pause the simulation.</u>** Drag the red movable line to a level a little bit above the mass. The red line will mark the point of release of the mass, to set the system in motion.
7. Measure the amplitude by measuring the distance between the green and red lines with the ruler tool.
8. Drag the mass up so the reference point is at the red line.
9. Bring in the stopwatch and press the Start button on the stopwatch. (It won't start counting yet because the simulation itself is paused.)



10. Step through the simulation frame-by-frame ()until the mass goes beyond the green line and returns to the red line. This represents one cycle of the mass-and-spring oscillation.
11. The measured period is the time on the stopwatch.
12. Use the period to find the "measured frequency". (Hint: Period and frequency are inverses of each other: $f = 1 / T$.)
13. Compare the two frequencies (the one from the spring constant, and the one from the period measurement) using a % Difference.

Equilibrium Stretch (cm)	
Spring Constant (N/m)	
Predicted Frequency (Hz)	
Amplitude (cm)	
Measured Period (s)	
Measured Frequency (Hz)	
% Difference of frequencies	

Table 1. Spring oscillation parameters obtained from the PhET Masses and Springs simulation, for an oscillator build with the default spring constant and a hanging mass of 100 grams.

Part#2. Period vs. Spring Constant

1. Move the Spring Constant slider for your spring all the way to the left. Use the same basic process from Part #1 to find the **spring constant and measured period**, with the 100 g mass but every other "notch" on the Spring Constant slider. Skip the steps from Part #1 that aren't necessary to obtain those two parameters. Don't make a Table 1 for every trial, just give the results in Table 2. Make sure to note the value of the mass in the caption.
2. Perform a trial for **every other notch of the Spring Constant slider**.
3. Calculate the squared frequency from the period, using $f^2 = (1/T)^2$.

Spring Constant Notch					
Equilibrium Stretch (cm)					
Spring Constant (N/m)					
Period (Hz)					
Squared Frequency (Hz^2)					

Table 2. Oscillation frequencies for several springs, each with the same suspended mass of ___kg.

Since the frequency is proportional to the square root of the spring constant, the frequency squared should be proportional to the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad\qquad f^2 = \frac{1}{4\pi^2 m}k$$

In a graph of f^2 vs. k, the slope should be equal to $(4\pi^2 m)^{-1}$

4. Graph Squared Frequency vs. Spring Constant. Fit the plot into a linear function and obtain the equation of the trendline.

5. Calculate $(4\pi^2 m)^{-1}$ and compare it with the slope of the graph.

$(4\pi^2 m)^{-1}$	
Slope	
% Difference	

Table 3. Analysis of …

Part #3. Period vs. Mass

1. Pick a single value of the spring constant and find **the period** with each available mass. Use a procedure similar to Part #2 Step 1 and 2 for gathering the data. Make sure to note which spring constant you used.
2. Calculate the squared period for each trial.
   Since the period is proportional to the square root of the mass, the period squared should be proportional to the mass.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = \frac{4\pi^2}{k}m$$

   In a graph of T^2 vs. m, the slope should equal $4\pi^2/k$.
3. For the first 3 trials (where you have a numerical mass), graph the Squared Period vs. Mass and find the slope of this graph.
4. Analyze the result (i.e. the slope) in a Table 3a, similar in style to Table 2a above.
5. Use the slope and squared period to calculate each mass.
   For the known masses, you can use the result to estimate the error of this process.
   For the unknown masses, you now have an estimate of the mass.

Suspended Mass (kg)	0.05	0.1	0.25	Pink	Blue	Orange
Period (s)						
Squared Period (s^2)						
Calculated Mass (kg)						

Table 3. Oscillation frequencies for several masses, each hanging from the ________ spring.

Part #4. Period vs. Amplitude

How should the period of oscillation change with the change of the amplitude? Perform a series of trials to investigate this. Use the mass and spring constant of your choice, and be sure to record them. In each of 5 trials, change the amplitude by moving the red line, and measure the amplitude and period to compare the trials.

Amplitude (cm)					
Period (s)					

Table 4. Oscillation period at different amplitudes for a mass of _______ kg and a spring constant of ______ N/m.

Requirements for the Report:

The report must contain a **Header** at the top (Title of Lab, Authors, and Date)

Abstract Section must contain the following in paragraph form:

- Brief Introduction that includes objectives and basic theory of the lab. Include:
    - What makes a system become an oscillator?
    - What quantities describe an oscillation?
    - What physical properties of an oscillator affect the oscillation?
- Methodology describing broadly what was done, using what tools, and what was measured/recorded.
- Data Summary including quantities worked into sentences.
    - *Part 1*: Investigate the data of period and frequency and discuss the relationship between the two variables. Compare the two methods of obtaining frequency.
    - *Part 2*: Investigate the data of spring constant and frequency. What is the relationship between spring constant and frequency? Does the data support the theoretical relationship? Use the graph and % difference to back up your analysis.
    - *Part 3*: Investigate the data of mass and period. What is the relationship between mass and period? Does the data support the theoretical relationship? Use the graph and % difference to back up your analysis. How close are the calculated masses to the known suspended masses? Are the calculated masses of the Pink, Blue, and Orange mass reasonable with respect to the known masses? Explain.
    - *Part 4*: Analyze the data and establish a relationship between amplitude and frequency. Is the relationship expected?
- Conclusions based on the quantitative results. Briefly summarize the results of each part of the lab from the Data Summary.
- Sources of Error and a ballpark estimate of their contribution. DO NOT use "human error". That term is too vague to be meaningful.

Data Section must contain the following:
[Each table and graph should be labeled and descriptively captioned. You can use the captions provided in the lab manual, but input the values used by your lab group.]

- 6 Tables
- 2 Graphs